Continuous Reactive-based Position-Attitude Control of Quadrotors

Anand Sanchez, Vicente Parra-Vega, Chinpei Tang, Fatima Oliva-Palomo and Carlos Izaguirre-Espinosa

***Abstract*— A new control scheme for position-orientation tracking of underactuated quadrotor robotic vehicle is proposed. A quaternion-based sliding surface parametrizes the open-loop error equation of orientation dynamics, then a second order sliding mode (SOSM) is synthesized for global exponential stabilization of attitude coordinates along an orientation equilibrium manifold. This SOSM for any initial condition leads to a simplified design of a torque *PD* controller for position dynamics, for globally uniformly ultimately bounded of position trajectories. The SOSM reacts to the effect of the PD as if it were an endogenous persistent disturbance, which vanishes until it reaches its equilibrium position manifold. In contrast to other results that consider the full model without linearization nor further simplifications, our proposal yields a controller which is smooth and does not require the dynamic model. Since the parametrization of attitude representation is global, aggressive maneuvering capabilities are exhibited. Simulations are presented for a variety of flight regimes, including carry out helixes and loops at high angular velocities. Real-time experiments provide a glimpse of the closed-loop performance for a custom made quadrotor.**

I. Introduction

It is acknowledge that the particular quadrotor exhibits characteristics that make then attractive as Unmanned Aerial Vehicle (UAV), such as small size, low weight, low consumption, and high maneuverability useful for a variety of applications. However, its particular flight dynamics also has shown high complexity that demands advanced control techniques to become a research area in its own right.

The quadrotor as flying 6-DoF rigid body robot, is modeled as an underactuated nonlinear system. It exhibits fast orientation dynamics coupled with slow position dynamics, which was exploited in [1] for an involved yet full control design. Certainly in the open, there is plenty of space to maneuver, and although most of the literature has been devoted to computed torque control for the stabilization of attitude, [2], [3], well-posed and robust position-orientation control is still a challenge. Being the quadrotor an underactuated system, backstepping technique has been applied, which are prone to instability for any uncertainties of the model, a likely case in practice, [4], [5]. The problem becomes more difficult when both position and orientation dynamics are targeted, [6], [7], which typically involves either solving angular velocities as function of position controller and thrust, based on full knowledge of dynamics and kinematic transformation. This assumes not only exact knowledge of dynamics, but also complex computations, including in some cases derivatives of the state. A separate discussion deserves the highly successful implementation of learning control, at the expense of trial-and-error [8], [9], even simple regulators have been proposed and tested, [10], including open-loop control [11]. However, learning or open-loop control may lead to catastrophic results in the first trials or hard to tune due to its overparametrized control.

To circumvent the problem of uncertain dynamics, the transformation and complex control structure, we develop in this paper a new second order sliding mode controller that enforces an integral sliding mode for any initial conditions of the orientation dynamics. The sliding surface depends on quaternion, consequently well posed for any rotation and stemmed on the fact that order reduction and invariance to system model arises, global well-posed orientation tracking is enforced. Then, position control is proposed as an additional torque input, it acts as an endogenous disturbance to the orientation controller, which reacts against it, but does alter its equilibrium position manifold. This allows a succinct approach to address the complex position-orientation tracking while avoiding the transformation, all these without using the dynamic model. As a result, our proposed control concept avoids computation of transformation of the relates position and orientation dynamics, guarantees global tracking of orientation while it enforces globally ultimately boundedness of position tracking errors.

Further, preliminary developments follows the direction of [7] for quaternion-based control, however a new sliding surface is proposed for a novel model-free SOSM. An immediate implication of this feature is the ability to carry out loops, based on singularity free quaternion representation. Remarkably, this controller can be synthesized by purely state feedback kinematic calculations since passivity-based control scheme is considered. Finally, our proposal ensures the *feasibility* of implementation, because it does not involve any non causal variable. Experimental results confirm this claim[1].

A. Organization

The control framework and stability analysis are presented in Section II. Section III describes representative scenarios in the dynamic simulation study, which illustrate the effectiveness of the control scheme. Preliminary representative

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experiments for a single quadrotor is presented in Section IV to show the effective fast and robust tracking features of the control scheme. Finally, we provide in Section V some concluding remarks.

II. Control Design and Stability Analysis

In this section, a Second Order Sliding Mode (SOSM) Controller is provided for the attitude system, [12]. Based on a quaternion representation of the attitude, the SOSM is synthesized, considering smooth desired signals and bounded disturbances. Then, globally uniformly ultimately bounded position trajectories are induced by applying a position-based torque.

The dynamic model of a Quadrotor is basically obtained representing the aircraft as a rigid body evolving in 3D and subject to one force and 3 moments. Let us consider earth fixed frame $\mathcal{I} = \{e_x, e_y, e_z\}$ and body fixed frame $\mathcal{A} = \{e_x^b, e_y^b, e_z^b\}$, as seen in Figure 1. The center of mass and the body fixed frame origin are assumed to coincide. The orientation of the rigid body is given by a rotation $\mathcal{R} : \mathcal{A} \longrightarrow \mathcal{I}$, where $\mathcal{R} \in SO(3)$ is an orthogonal rotation matrix. Newtons-Euler equations of motion state the dynamics of the quadrotor as follows:

$$\begin{aligned} m\ddot{\xi} &= -\mathcal{T}\mathcal{R}e_z + F(t) && (1)\\ \dot{\mathcal{R}} &= \mathcal{R}[\Omega\times] && (2)\\ \mathbf{J}\dot{\Omega} &= -\Omega \times \mathbf{J}\Omega + \tau + d(t) && (3) \end{aligned}$$

where $\xi = (x, y, z)^T$ denotes the position of the center of mass of the airframe in the frame $\mathcal{I}$ relative to a fixed origin, $\Omega = (\Omega_1, \Omega_2, \Omega_3)^T \in \mathcal{A}$ denotes the angular velocity of the airframe expressed in the body fixed frame. m denotes the mass of the rigid object and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ denotes the constant inertia matrix around the center of mass (expressed in the body fixed frame $\mathcal{A}$). $[\Omega\times]$ denotes the skew-symmetric matrix of the vector Ω. $\mathcal{T} \in \mathbb{R}_+$ represents the magnitude of the principal non-conservative forces applied to the object. $F(t)$ represents the external forces applied to the aerial vehicle, such that in the absence of forces exerted by the ambient (aerodynamic reaction forces, etc.) $F(t) = mge_z$. $\tau \in \mathcal{A}$ is the control torque. $d(t) \in \mathbb{R}^3$ represents the external torque disturbances induced by $F(t)$.

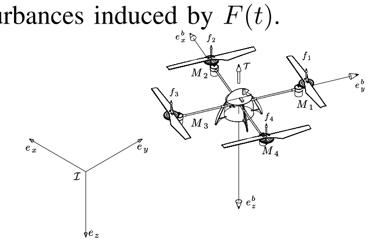

Fig. 1. The UAV system. f_i represents the thrust of motor M_i and $\mathcal{T}$ is the main thrust.

A. Attitude Control

We employ the unit quaternion as the attitude representation. Using this representation the attitude control design does not suffer from singularities. The unit quaternion is defined as

$$\mathbf{q} = \begin{pmatrix} q_0 \\ q \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\mu}{2}\right) \\ e\sin\left(\frac{\mu}{2}\right) \end{pmatrix} \quad (4)$$

where e is the Euler axis and μ is the Euler angle. The unit quaternion satisfies the following constraint

$$\mathbf{q}^T\mathbf{q} = q_0^2 + q^Tq = 1 \quad (5)$$

and it is related to the angular velocity Ω by the following differential equations

$$\dot{\mathbf{q}} = \begin{pmatrix} \dot{q}_0 \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}q^T\Omega \\ \frac{1}{2}\left(q_0\Omega + [q\times]\Omega\right) \end{pmatrix} \quad (6)$$

Let us define the angular velocity error Ω_e as follows

$$\Omega_e = \Omega - \Omega_d \quad (7)$$

where Ω_d is the desired angular velocity expressed in the body fixed frame.

We now present the control methodology of the SOSM. Let us define a parametrization Y_r in terms of a nominal references Ω_r, to be defined, and its derivative $\dot{\Omega}_r$, as follows

$$Y_r = \mathbf{J}\dot{\Omega}_r + [\Omega\times]\mathbf{J}\Omega_r + [\Omega_r\times]\mathbf{J}\Omega - [\Omega_r\times]\mathbf{J}\Omega_r \quad (8)$$

Introducing (8) into (3), and defining $\tau \triangleq \tau_a + \tau_p$, yields

$$\mathbf{J}\dot{S}_r + [S_r\times]\mathbf{J}S_r = \tau_a + \tau_p + d(t) - Y_r \quad (9)$$

where the error coordinates S_r are defined by

$$S_r = \Omega - \Omega_r \quad (10)$$

and τ_a, and τ_p denote the attitude torque input and the position-based torque (to be defined later), respectively.

Let us consider the following nominal reference Ω_r

$$\begin{aligned} \Omega_r &= \Omega_d - \alpha q_e + S_d - \gamma\sigma && (11)\\ \dot{\sigma} &= \operatorname{sgn}(S_q) && (12) \end{aligned}$$

where feedback gains, $\alpha > 0$ and γ is diagonal positive definite matrix, the function $\operatorname{sgn}(x) = (\operatorname{sgn}(x_1), \operatorname{sgn}(x_2), \operatorname{sgn}(x_3))^T$ stands for the input wise discontinuous function of x, and

$$\begin{aligned} S_q &= S - S_d && (13)\\ S &= \Omega_e + \alpha q_e && (14)\\ S_d &= S(t_0)\exp(-k(t - t_0)) && (15) \end{aligned}$$

for $k > 0$ and $S(t_0)$ stands for $S(t)$ at $t = t_0$. $\mathbf{q}_e = (q_{0e}, q_e^T)^T$ is the relative attitude error with q_e given by [7]

$$q_e = -q_0q_d + q_{0d}q - [q\times]q_d \quad (16)$$

where $\mathbf{q}_d = (q_{0d}, q_d^T)^T$ is the desired attitude, such that $q_{0d}(t), q_d(t)$ are one time differentiable functions. Notice that $S_q(t_0) = 0$ for any initial condition. From (7), (10), (11) and (14) the dynamic error coordinates S_r are given by

$$S_r = S_q + \gamma\sigma \quad (17)$$

being S_q the sliding surface. Let us now introduce some structural properties of (3) and (11), which will be used in the stability analysis.

1) properties: There exist positive scalars β_i for $i = 0, \ldots, 4$, such that

$$\begin{array}{r}0 < \beta_0 < \lambda_{\min}(\mathbf{J}) \leq \|\mathbf{J}\| \leq \lambda_{\max}(\mathbf{J}) < \beta_1 < \infty \\ \|q_e\| < 1 \\ \|\Omega_r\| \leq \beta_2 + \|\gamma\|\|\sigma\| \\ \|\dot{\Omega}_r\| \leq \beta_3 + \beta_4\|\Omega_e\|\end{array} \tag{18}$$

where $\lambda_{\min}(\mathbf{J}), \lambda_{\max}(\mathbf{J})$ stand for the minimum and maximum eigenvalues of matrix $\mathbf{J} \in \mathbb{R}^3$, $\|\mathbf{J}\| = \sqrt{\lambda_{\max}(\mathbf{J}^T\mathbf{J})}$ and $\|\cdot\|$ stands for the vector Euclidean norm.

The disturbance $d(t)$ and the position-based torque τ_p are assumed to be bounded. From (8), (3) and using (18), $\Gamma \triangleq \tau_p + d(t) - Y_r$ can be bounded as

$$\begin{aligned}\|\Gamma\| \leq\ & \|\tau_p\| + \|d(t)\| + \|\mathbf{J}\|\|\dot{\Omega}_r\| \\ & + 2\|\Omega\|\|\mathbf{J}\|\|\Omega_r\| + \|\mathbf{J}\|\|\Omega_r\|^2 \\ \leq\ & \|\tau_p\| + \|d(t)\| + \beta_1\beta_3\|\Omega_e\| + 2\beta_1\beta_2\|\sigma\|\|\Omega\| \\ & + 2\beta_1\beta_2\|\gamma\|\|\sigma\| + \beta_1\|\gamma\|^2\|\sigma\|^2 + \beta_5 \\ \leq\ & c_p + \eta(t)\end{aligned} \tag{19}$$

where $\beta_5 = \beta_1\beta_4 + \beta_1\beta_2^2$, $\|\tau_p\| \leq c_p$ for some $c_p > 0$ and $\eta(t)$ is a state-dependent function. Notice that, $\eta(t)$ considers all the external torques including state-dependence of $d(t)$.

Consider the following control law

$$\tau_a = -K_d S_r \tag{20}$$

where K_d is a diagonal positive definite matrix. We now have the following result.

Theorem 1: Consider the attitude dynamics (3) in closed-loop with the controller (20). Then, semiglobal exponential tracking is assured, provided that γ in (17) and K_d are large enough, for small initial errors.

B. Stability Analysis

Proof: [Proof of Theorem 1] Substituting (20) into (9) yields

$$\mathbf{J}\dot{S}_r = -\left(K_d S_r + [S_r\times]\mathbf{J}S_r\right) + \tau_p + d(t) - Y_r \tag{21}$$

Let us consider the following Lyapunov function

$$V = \frac{1}{2}S_r^T\mathbf{J}S_r \tag{22}$$

The total derivative of (22) along its solution (21) gives rise to

$$\dot{V} = -S_r^T K_d S_r + S_r^T\left(\tau_p + d(t) - Y_r\right) \tag{23}$$

Since K_d is a diagonal positive definite matrix, then $S_r^T K_d S_r$ satisfies

$$\lambda_{\min}(K_d)\|S_r\|^2 \leq S_r^T K_d S_r \leq \lambda_{\max}(K_d)\|S_r\|^2 \tag{24}$$

Using (19) and (24), (23) becomes

$$\dot{V} \leq -\|S_r\|\left(\lambda_{\min}(K_d)\|S_r\| - c_p - \eta(t)\right) \tag{25}$$

Let $c = \sup_{t\geq 0}\eta(t)$. Note that if $\|S_r\| > [(c_p + c)/\lambda_{\min}(K_d)]$ then $\dot{V} < 0$. This implies that exists a time t_1 such that

$$\|S_r\| \leq c_T \quad \forall t > t_1 \tag{26}$$

where $c_T = (c_p + c)/\lambda_{\min}(K_d)$. In this way S_r is upper bounded by $c_T/\lambda_{\min}(K_d)$. Boundedness of S_r implies the boundedness of the state which includes σ. Therefore, we can conclude the boundedness of $\dot{S}_r$, from (21), as follows

$$\begin{aligned}\|\dot{S}_r\| \leq\ & \lambda_{\max}(\mathbf{J}^{-1})\left[\left(\lambda_{\max}(K_d) + c_T\lambda_{\max}(\mathbf{J})\right)c_T + c_p + c\right] \\ \leq\ & \bar{c}\end{aligned} \tag{27}$$

Now, for a given γ, sliding mode is induced on $S_q = 0$. Consider the following dynamical system from (17)

$$\dot{S}_q = -\gamma\,\mathrm{sgn}(S_q) + \dot{S}_r \tag{28}$$

with the following positive definite function

$$V_q = \frac{1}{2}S_q^T S_q \tag{29}$$

The total derivative of (29) , along its solution (28) gives rise to

$$\begin{aligned}\dot{V}_q = -S_q^T\gamma\,\mathrm{sgn}(S_q) + S_q^T\dot{S}_r \leq\ & -\lambda_{\min}(\gamma)|S_q| + |S_q||\dot{S}_r| \\ \leq\ & -\nu|S_q|\end{aligned}$$

where $\nu = \lambda_{\min}(\gamma) - \sqrt{3}\bar{c}$ and $|\cdot|$ stands for the vector norm-1.

Thus, in order to prove that $S_q \to 0$ in finite time, $\nu > 0$ is chosen for guaranteing the existence of a sliding mode condition $S_q^T\dot{S}_q \leq -\nu|S_q|$. This implies that a sliding mode is established at time $t_s \leq |S_q(t_0)|/\nu$, and since $S_q(t_0) = 0$ for any initial condition, then the sliding mode in $S_q(t) = 0$ is enforced for all time.

Considering that $S_d \approx 0$, for a tuned large enough k (this is achieved numerically in few samplings), and since $S_q = 0$ for all time, then

$$\Omega_e = -\alpha q_e \quad \forall t > t_1 \tag{30}$$

According to (6), the time derivative of $\mathbf{q}_e$ is given by

$$\dot{\mathbf{q}}_e = \begin{pmatrix} -\frac{1}{2}q_e^T\Omega_e \\ \frac{1}{2}\left(q_{0e}I + [q_e\times]\right)\Omega_e \end{pmatrix} \tag{31}$$

Then, from (30) it gets

$$\dot{q}_{0e} = \frac{\alpha}{2}f_1(t) \tag{32}$$

$$\dot{q}_e = -\frac{\alpha}{2}q_{0e}q_e \tag{33}$$

where $f_1(t) = q_e^T q_e$. Note that, $f_1(t)$ is a positive definite function of q_e. Then the solution of (32), for $t > t_1$, is given by

$$q_{0e}(t) = q_{0e}(t_1) + \frac{\alpha}{2}\int_{t_1}^{t} f_1(\zeta)d\zeta \tag{34}$$

Assuming, without loss of generality, that $q_{0e}(t_1) > 0$. Then we can conclude that

$$q_{0e}(t) > 0 \quad \forall t > t_1 \tag{35}$$

The solution of (33), for $t > t_1$, is given by

$$q_e(t) = e^{-\frac{\alpha}{2}q_{0e}(t)(t-t_1)}q_e(t_1) \tag{36}$$

for $t > t_1$. From (35), we conclude that

$$q_e(t) \to 0 \tag{37}$$

exponentially. From the constraint of a unit quaternion (5), it follows that $q_{0e} \to 1$ exponentially. This completes the proof of Theorem 1. ■

C. Position Control

Consider the following position-based torque

$$\tau_p = \begin{pmatrix} -k_1\dot{y} - k_2(y - y_d) \\ \bar{k}_1\dot{x} + \bar{k}_2(x - x_d) \\ 0 \end{pmatrix} \tag{38}$$

where y_d and x_d are the desired positions, k_1, k_2, $\bar{k}_1$ and $\bar{k}_2$ are positive constants.

From (26) and (27), we have shown that the trajectories of S_r and $\dot{S}_r$ remain bounded for $t > t_1$. Let us define $(\delta_1(t), \delta_2(t), \delta_3(t))^T$, from (21), as

$$\begin{pmatrix} \delta_1(t) \\ \delta_2(t) \\ \delta_3(t) \end{pmatrix} = \mathbf{J}\dot{S}_r + [S_r \times]\mathbf{J}S_r + K_d S_r - d(t) + Y_r \tag{39}$$

Since both S_r and $\dot{S}_r$ remain bounded, in view of (19) and by considering the case of attitude regulation $\Omega_d = 0, \mathbf{q}_d = (1,0,0,0)^T$, it follows that there exists positive constants c_1, c_2, c_3 such that for $t > t_1$,

$$|\delta_1(t)| \le c_1, \quad |\delta_2(t)| \le c_2, \quad |\delta_3(t)| \le c_3 \tag{40}$$

Substituting (38) and (39) into (21), yields the following differential equations

$$-k_1\dot{y} - k_2(y - y_d) = \delta_1(t) \tag{41}$$

$$-\bar{k}_1\dot{x} - \bar{k}_2(x - x_d) = -\delta_2(t) \tag{42}$$

Note that, the control input (41) is acting on the dynamics of y through a negative moment around $x - axis$ and (42) is acting on the dynamics of x through a positive moment around $y - axis$ (see figure 1).

Now, from (1), the dynamical equation of the altitude z is given by

$$m\ddot{z} = -\mathcal{T} r_{33} + f_3(t) \tag{43}$$

where r_{33} denotes the entry at the third row and third column of matrix $\mathcal{R}$ and $f_3(t)$ denotes the third component of the vector $F(t)$.

Consider the altitude control, through thrust $\mathcal{T}$, as follows

$$\mathcal{T} = m\left[l_1\dot{z} + l_2(z - z_d) + g\right] \tag{44}$$

where z_d is the desired altitude and l_1 and l_2 are positive constants such that the polynomial $s^2 + l_1 s + l_2$ is stable. Substituting (44) into (43) yields

$$m\ddot{z} = -m\left[l_1\dot{z} + l_2(z - z_d) + g\right] r_{33} + f_3(t) \tag{45}$$

In view of the attitude stability derived from Theorem 1, and considering that in general $f_3(t) \approx mg$ it follows that

$$\begin{aligned} r_{33} &\to 1 \\ f_3(t)/m - g = \bar{f}_3(t) &\le c_g \end{aligned} \tag{46}$$

for some $c_g > 0$ small enough.

Let us now define the following position tracking error

$$\begin{aligned} \tilde{y} &= y - y_d \\ \tilde{x} &= x - x_d \\ \tilde{z} &= (z - z_d, \dot{z} - \dot{z}_d)^T \end{aligned} \tag{47}$$

Tacking the time derivative of (47) and substituting for equations (42), (41), (45), gives rise to

$$\begin{aligned} \dot{\tilde{y}} &= -\tfrac{k_2}{k_1}\tilde{y} - \tfrac{\delta_1(t)}{k_1} - \dot{y}_d \\ \dot{\tilde{x}} &= -\tfrac{\bar{k}_2}{\bar{k}_1}\tilde{x} + \tfrac{\delta_2(t)}{\bar{k}_1} - \dot{x}_d \\ \dot{\tilde{z}} &= A\tilde{z} + B\left(-l_1\dot{z}_d - \ddot{z}_d + \bar{f}_3(t)\right) \end{aligned} \tag{48}$$

where

$$A = \begin{pmatrix} 0 & 1 \\ -l_2 & -l_1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

such that A is a Hurwitz matrix.

By assuming that the time derivatives of the desired positions are bounded

$$|\dot{y}_d| \le c_{yd}, \quad |\dot{x}_d| \le c_{xd}, \quad |\dot{z}_d| \le c_{zd}, \quad |\ddot{z}_d| \le c_{zpd} \tag{49}$$

for some positive constants c_{yd}, c_{xd}, c_{zd}, and c_{zpd}, then the solution of (48) is given by

$$\begin{aligned} \tilde{y} &= e^{-\frac{k_2}{k_1}(t-t_0)}\tilde{y}(t_0) - \int_{t_0}^{t} e^{-\frac{k_2}{k_1}(t-\tau)}\left(\tfrac{\delta_1}{k_1} + \dot{y}_d\right)d\tau \\ \tilde{x} &= e^{-\frac{\bar{k}_2}{\bar{k}_1}(t-t_0)}\tilde{x}(t_0) + \int_{t_0}^{t} e^{-\frac{\bar{k}_2}{\bar{k}_1}(t-\tau)}\left(\tfrac{\delta_2}{\bar{k}_1} - \dot{x}_d\right)d\tau \\ \tilde{z} &= e^{A(t-t_0)}\tilde{z}(t_0) - \int_{t_0}^{t} e^{A(t-\tau)}B\left(l_1\dot{z}_d + \ddot{z}_d - \bar{f}_3\right)d\tau \end{aligned} \tag{50}$$

Using (40), (46) and (49), (50) becomes

$$\begin{aligned} |\tilde{y}| &\le e^{-\frac{k_2}{k_1}(t-t_0)}|\tilde{y}(t_0)| + \tfrac{1}{k_2}\left(c_1 + c_{yd}k_1\right) \\ |\tilde{x}| &\le e^{-\frac{\bar{k}_2}{\bar{k}_1}(t-t_0)}|\tilde{x}(t_0)| + \tfrac{1}{\bar{k}_2}\left(c_2 + c_{xd}\bar{k}_1\right) \\ \|\tilde{z}\| &\le \kappa e^{-a(t-t_0)}\|\tilde{z}(t_0)\| + \tfrac{\kappa}{a}\left(l_1 c_{zd} + c_{zdp} + c_g\right) \end{aligned} \tag{51}$$

where constants $\kappa > 0$ and $a > 0$ are such that $\|e^{A(t-t_0)}\| \le \kappa e^{-a(t-t_0)}$.

In view of (51), and for

$$|\tilde{y}(t_0)| < a_1, \quad |\tilde{x}(t_0)| < a_2, \quad \|\tilde{z}(t_0)\| < a_3$$

with arbitrarily large positive constants a_1, a_2, a_3, then the system (48) is globally uniformly ultimately bounded for all $t \ge t_0 + t_1$.

III. Simulations Results

Two representative scenarios are discussed: regulation, tracking with aggressive maneuvering.

A. The quadrotor simulator system

A custom made quadrotor is considered, with a mass of $m = 0.422Kg$ and a tensor of inertial matrix computed with a CAD model,

$$\mathbf{J} = \begin{bmatrix} 2.098 & 63577.538 & -2002.648 \\ 63577.538 & 2.102 & 286.186 \\ -2002.648 & 286.186 & 4.068 \end{bmatrix} Kg \times m^2$$

values are $\times 10^{-9}$, except the first and second entry of the diagonal that are $\times 10^{-3}$. Motors are located symmetrically at $22.5cm$ *wrt* to the centroid, located at the origin measured with the balancing method. It is assumed that thrust map is available, then controller τ can be directly programmed. The simulator is programmed in Matlab with a fixed step numerical integrator $RK8$ $1ms$.

B. Regulation Case

Desired regulation point is $(x_d, y_d, z_d)^T = (0.1, 0.1, 0.5)^T m$, with initial conditions at $x(t_0), y(t_0), z(t_0) = (0, 0, 0)mt$. Desired orientation angles are set to zero. Feedback gains to stabilize the system are $K_d = diag(2, 2, 2)$, sliding mode gain is $\gamma = diag(0.001, 0.001, 0.001)$, with $\alpha = 4$. Feedback gains for the PD position control are $k_1 = \bar{k}_1 = 8, k_2 = \bar{k}_2 = 6, l_1 = 2, l_2 = 1$.

Figure 2 shows fast yet smooth regulation in position coordinates, without overshoot. Control effort can be seen in Figure 4, where it responds accordingly to disturbances of position control. Convergence to zero of orientation errors can be seen in terms of quaternion in Figures 3, which implies global regulation of its corresponding orientation angles.

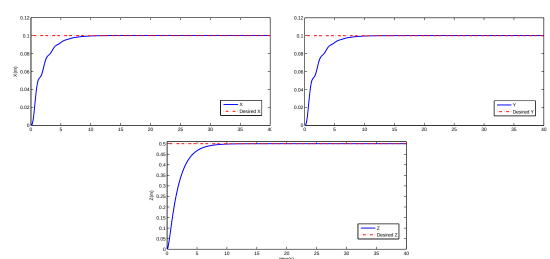

Fig. 2. Simulation: Position during regulation

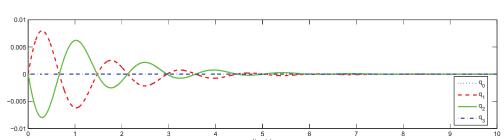

Fig. 3. Simulation: Zoom of the quaternion, it remains around zero for regulation. q_0 is also around 1 (not shown in this scale).

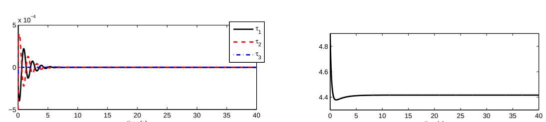

Fig. 4. Simulation: Torque and thrust for regulation.

C. Tracking Case

A loop is programmed using the same feedback gains as those in subsection III-B. In this case, the quadrotor is elevated from ground to $z_d = 10mt$ in $14s$, with $x_d(t) = x(t_0)$ and $x_d(t) = x(t_0)$, while zeroing orientation angles. Then, position control is turned off to start looping at $5Hz$ (a loop is completed at $0.2s$), during $0.8s$, then orientation is zeroing again and height is regulated at $z_d = 6mt$, see Figure 5. Figure 6 indeed shows a very aggressive maneuver, a loop where the quadrotor frame flipping over and over, thanks to the well posed quaternion representation and the fast and robust controller. The reaction strategy is clearly appreciated in terms of the sliding surface, see Figure 7.

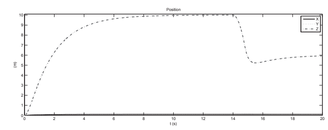

Fig. 5. Simulation: Controlled height during the loop regime. x, y coordinates are regulated at zero.

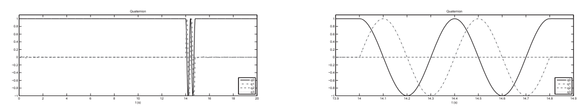

Fig. 6. Simulation: This extreme flight regime takes the control proposal to the limit. Quaternion show a complete flip over when it goes from $+1$ to -1 in $200ms$ while falling down, carrying out 4 loops in $0.8s$. Figure at the bottom is a zoom during looping.

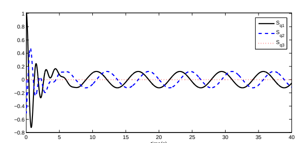

Fig. 7. Simulation: Quasi-sliding mode is maintained as a result of the reaction of the orientation controller (notice that ideal sliding mode occurs at $S_q = 0$).

D. Remarks

The closed-loop exhibits a highly maneuverable quadrotor, with the orientation dynamics reacting rather than rejecting the position control. A direct consequence of this scheme that deals with simultaneous position-orientation stabilization is the ability to perform aggressive maneuvering like ascending in helix, with increasingly amplitude, and extreme ones, like descending looping over and over (the last case is presented here). The instantaneous effect of the endogenous disturbance (the position controller), does not destabilize the system because the equilibrium manifold of orientation dynamics is independent of the equilibrium (manifold) of the such endogenous disturbance. In this way, orientation manifold $S_q = 0$ is indeed perturbed and it remains within the domain of attraction of the sliding mode so as to recover.

IV. EXPERIMENTAL RESULTS

The platform consists of an aerial vehicle (four-rotor rotorcraft), a ground station and a vision system (conformed by 7 cameras facing towards a mutual interest area).

A. Aerial vehicle

The distance between rotational axis of two rotors along same in the same axis is $36cm$,, mass is $0.5Kg$. A Li-Po battery feeds the embedded system and the four brushless motors are controlled with PWM signals tuned at $1ms$ to $2ms$, at a frequency of 496Hz. An 8-bit microprocessor

RABBIT RCM3400 module running at 29.4 MHz, handles the control unit, motors speed controller, sensors, as well as XBee ZB ZigBee PRO radio modem in both sides (quadrotor and ground station) at 2.4 GHz IEEE 802.15.4. The ground station runs Linux Debian OS R5.0 Lenny, kernell 2.6.30.5 RTAI. A 16 bits flight simulator joystick sends data generated by the user at 100 Hz. A Vision MX T-series with Tracker software obtains the translational position, angular position, translational velocity and angular velocity at 75 Hz.

B. Results

Task is tracking a circle of diameter of $1m$, at a frequency of $1.5Hz$ in the plane (x, y). Tracking is achieved, with smooth control activity, while angles of airframe remains around zero, see Figures 8-10.

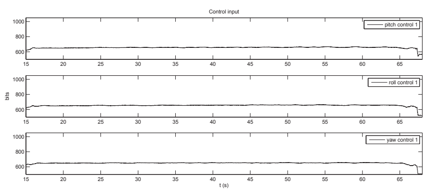

Fig. 8. Experiment: Controller torques, in counts.

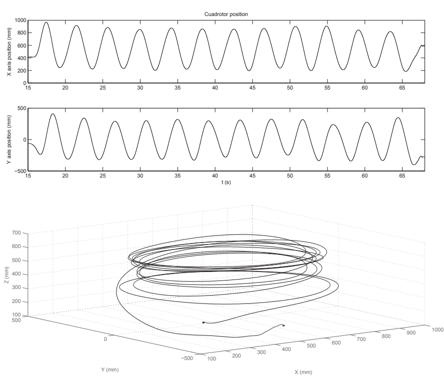

Fig. 9. Experiment: Position tracking of a circle at $1m$, in counts (above), and 3D view of the circle (below).

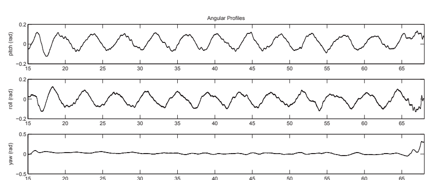

Fig. 10. Experiment: Angles of airframe.

V. CONCLUSIONS

A novel control scheme for quadrotors is proposed and explores the notion of reaction torque to simplify the design, yet delivers tracking of the full nonlinear model. An extended error parametrizes the open-loop error equation so as to SOSM enforces an sliding mode for all time and any initial condition. This very fact guarantees convergence of generalized coordinates of orientation. Additionally, a torque *PD* controller is proposed that introduces stability of position trajectories without solving the transformation that relates rotational matrix to angular velocities. In this sense, the SOSM reacts to the effect torque effect of the PD control, which in turns induces an equilibrium manifold for position dynamics. Simulations are presented, in particular for aggressive maneuvers. Representative real-time experiments for tracking show the effectiveness of the proposed scheme.

ACKNOWLEDGEMENTS

Authors gratefully and heartily acknowledges the collaboration with Prof. Mark Spong and his disposition to allow us to carry out experimental work in his Laboratory of Autonomous Robotic Systems, of The Erik Johnson School of Engineering & Computer Science University of Texas at Dallas, USA. Support is acknowledged also from Research Projects #133544, #133346 and #98265 for Fundamental Research, from the National Council of Science and Technology –Conacyt–, Mexico

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